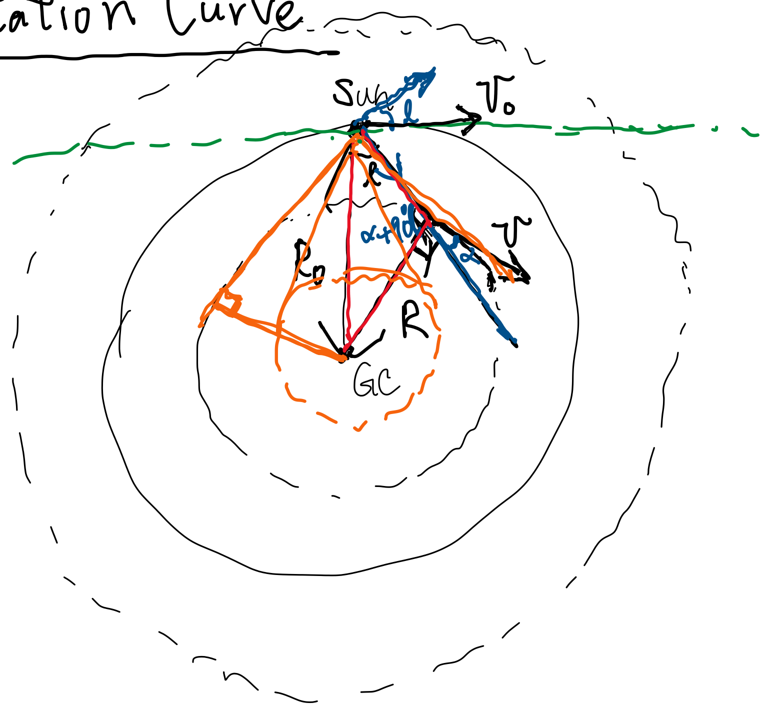
Week 3 Discussion

Wednesday, April 15, 2020 1:44 PM

Rotation Curve



$$V_r = V\cos\alpha - V_0 \sin l \quad — *$$

$$\frac{R}{\sin l} = \frac{R_0}{\sin(\alpha+90°)} = \frac{R_0}{\cos\alpha}$$

$$* \Rightarrow V_r = V\left(\frac{R_0 \sin l}{R}\right) - V_0 \sin l$$

$$= \boxed{R_0 \sin l \left(\frac{V}{R} - \frac{V_0}{R_0}\right)}$$

* longtitude range

limit for inner ring : $l_{max} > l > -l_{max}$

where $l_{max} < 90°$

outer ring : $360° > l > 0°$

$\left[\begin{array}{l} 270^\circ > l > 90^\circ : \text{purely outer.} \\ 90^\circ > l > -90^\circ : \text{inner + outer.} \end{array}\right.$

Given $V = \text{const}$ $(200-220\ \text{km/s})$.

$$V_r = R_0 \sin l \left(\frac{V}{R} - \frac{V_0}{R_0} \right)$$

$$= R_0 \sin l \times V \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$= \boxed{V \sin l \left(\boxed{\frac{R_0}{R}} - 1 \right)}$$

(i) Inner ring: ① $R < R_0 \Rightarrow \left(\frac{R_0}{R} - 1 \right) > 0$

② $l_{max} > l > -l_{max}$

$\Rightarrow$ $0^\circ > l > -l_{max} \Rightarrow \sin l < 0$ — $V_r < 0$

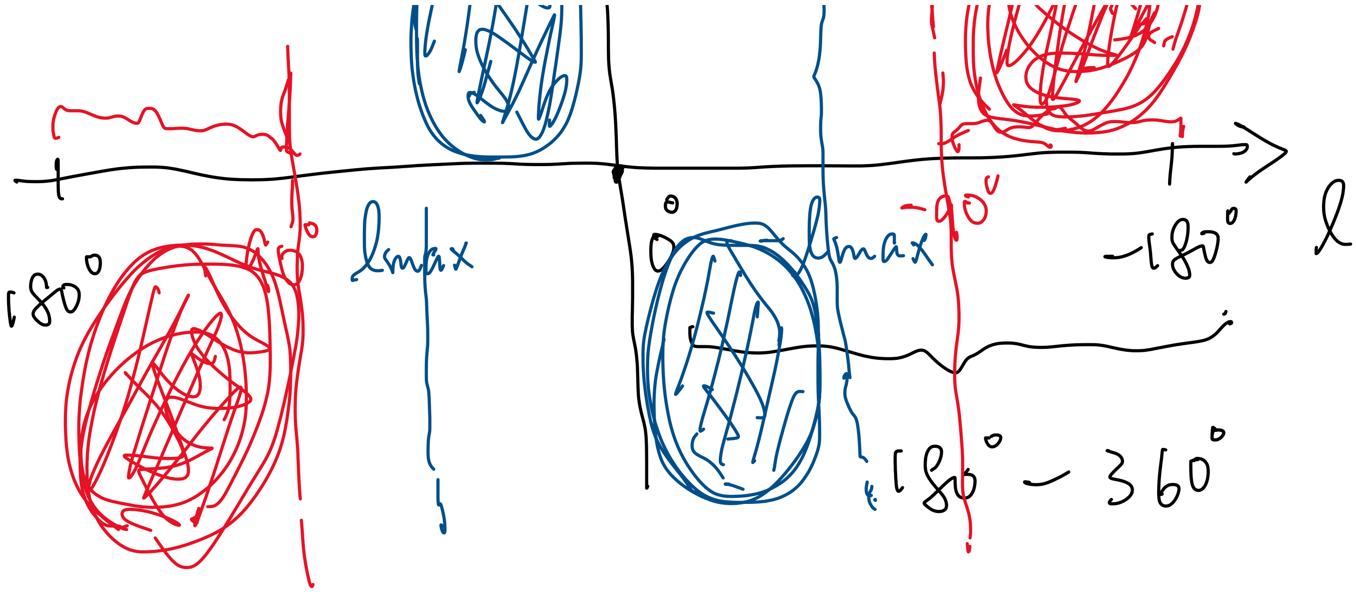
$l_{max} > l > 0^\circ \Rightarrow \sin l > 0$ — $V_r > 0$

(ii) Outer ring: ① $R > R_0 \Rightarrow \left(\frac{R_0}{R} - 1 \right) < 0$

② $360^\circ > l > 0^\circ$

$\Rightarrow$ $180^\circ > l > 0^\circ \Rightarrow \sin l > 0$ — $V_r < 0$

$360^\circ > l > 180^\circ \Rightarrow \sin l < 0$ — $V_r > 0$



180°
90°
lmax
0°
lmax
-90°
-180°
l
180° - 360°