Week 4 Discussion

Wednesday, April 22, 2020 1:57 PM

Virial Theorem

virial eq. $\underbrace{2T}_{\text{kinetic energy}} + \underbrace{V}_{\text{potential energy}} = 0$ — $*$.

→ mass of a gravitationally bound object

$\boxed{\text{Table 3.1}}$

M4 cluster: $\begin{cases} \sigma_r \approx 4 \ \text{km/s} \\ r_c \approx 0.5 \ \text{pc} \\ m = 6 \times 10^4 M_\odot \end{cases}$

$$T = \frac{1}{2} m \sigma_v^2 = \frac{3}{2} m \sigma_r^2$$

$$V = -\frac{Gm^2}{2r_c}$$

$$* \Rightarrow \cancel{2}\left(\frac{3}{\cancel{2}} \cancel{m} \sigma_r^2\right) = \frac{Gm^{\cancel{2}}}{2r_c}$$

$$\Rightarrow 3\sigma_r^2 = \frac{Gm}{2r_c} \Rightarrow \boxed{m = \frac{6\sigma_r^2 \cdot r_c}{G}}$$

$\sigma_r = 4 \ \text{km/s}, \ r_c = 0.5 \ \text{pc}$ ← convert to SI.

$G \doteq 6.67 \times 10^{-11}$ (SI unit)

$$\therefore \quad 6(\underbrace{4000}_{\text{m/s}})^2(0.5 \times \underbrace{3.086 \times 10^{16}}_{\text{pc to m}})$$

$m \simeq \dfrac{\qquad}{6.67 \times 10^{-11}}$

$\simeq 2.22 \times 10^{34}$ (kg)

$\simeq \dfrac{2.22 \times 10^{34} \text{ kg}}{2 \times 10^{30} \text{ kg}} \sim \underline{10^{4} M_{\odot}}$

$\rightarrow m = \underline{6 \times 10^{4} M_{\odot}}$

Conversion between $\underbrace{\rho}_{\text{density distribution}}$ and $\underbrace{\phi}_{\text{gravitational potential.}}$

HW: given $\phi \Rightarrow$ obtain ρ

poisson's eq: $\underline{\nabla^2 \phi = 4\pi G \rho}$.

given $\rho \Rightarrow$ obtain ϕ.

(Problem 3.3 in textbook).

"dark matter halo" density.

Given ρ $\quad$ $\rho(r_0)$

Given $\rho_{sis} = \dfrac{\ \ }{(r/r_0)^2}$.

Show that $\phi_{sis} = V_H^2 \ln\left(\frac{r}{r_0}\right)$

where $V_H = 4\pi G r_0^2 \rho(r_0)$

Spherical density distribution

(HW3 problem I, $a_H \to 0$)

$$\phi(r) \sim -\frac{GM}{r}$$

$$\Rightarrow -\int \frac{G\rho(r)}{\cancel{r}} \cdot \overbrace{4\pi r^{\cancel{2}}\, dr}^{dV}$$

$$\therefore \phi_{sis}(r) = -\int_r^{r_0} 4\pi G \rho_{sis}(r) \cdot r\, dr$$

$$= -\underbrace{4\pi G \cdot r_0^2 \rho(r_0)}_{V_H} \underbrace{\int_r^{r_0} \frac{1}{r^{\cancel{2}}} \cdot \cancel{r}\, dr}$$

$$= -V_H \cdot \ln\left(\frac{r_0}{r}\right)$$

$$= V_H \cdot \ln\left(\frac{r}{r_0}\right)$$