Midterm Review

1) Luminosity, Flux, Magnitude
2) Angular size ⟷ linear size
3) Virial theorem
4) Mass functions.

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1) Given Sun: $T_{eff} \approx 5780$ K

$R_\odot \approx 695700$ km

i) solar luminosity?

$$\underbrace{I}_{\boxed{\text{energy flux}}} = \frac{L}{4\pi R^2} = \sigma_{SB} \cdot T_{eff}^4$$

($E/t \cdot A$)

$$\Rightarrow L_\odot = \sigma T^4 \cdot 4\pi R_\odot^2 \approx 3.85 \times 10^{33}\ \text{erg/s}$$

(ii) flux observed from Earth?

$$F_\odot = \frac{L_\odot}{4\pi d^2} = \frac{\cancel{4\pi} R^2 \cdot \sigma T^4}{\cancel{4\pi}\, d^2} \approx 1.37 \times 10^6\ \frac{\text{erg}}{\text{s} \cdot \text{cm}^2}$$

(iii) If $\begin{bmatrix} L_{vega} = 40 L_\odot \\ d_{vega} = 7.68\,\text{pc} \end{bmatrix}$,

apparent magnitude of Sun ($m_\odot$)

(Vega system)

① $m_1 - m_2 = -2.5 \log\left(\frac{F_1}{F_2}\right)$

② $m - M = 5 \log(d) - 5$

$d_{vega} = 7.68$ pc

$$① \Rightarrow \underbrace{m_\odot - m_{vega}}_{= 0} = -2.5 \log\left(\frac{L_\odot \cdot \cancel{4\pi} \boxed{d_{vega}^2}}{\cancel{4\pi} \underbrace{d_\odot^2}_{1\,AU} \cdot \underbrace{L_{vega}}_{40 L_\odot}}\right)$$

1 AU → pc?

$\Rightarrow m_\odot \approx -27.75$ mag [actual mag varies by band].

(−26.8)

(iv) absolute magnitude of Sun ($M_\odot$)?

$\rightarrow m_\odot - M_\odot = 5 \log(d) - 5$

$$\underbrace{d}_{1\,AU} = \frac{1}{206265}\,\text{pc} \approx 4.85 \times 10^{-6}\ \text{pc}$$

$\Rightarrow M_\odot \approx 6$ mag

(4.86)

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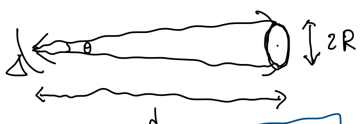
2) Angular sizes of $\begin{bmatrix} \text{Sun} \\ \text{Moon} \end{bmatrix}$ as viewed from Earth?

* objects far way

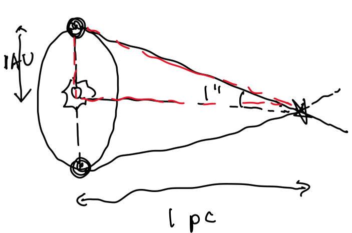
→ small angle approximation.



$$2R \approx d \cdot \theta \Rightarrow \boxed{\theta = \frac{2R}{d}}$$

$\begin{cases} R_\odot = 695700\ \text{km} \\ d_\odot = 1.5 \times 10^8\ \text{km} \end{cases}$ $\begin{cases} R_{moon} = 1737\ \text{km} \\ d_{moon} = 384{,}399\ \text{km} \end{cases}$

$\begin{cases} \theta_\odot \approx 0.01\ \text{rad} \approx 35' \\ \theta_{moon} \approx 0.01\ \text{rad} \approx 35' \end{cases}$ total eclipses are possible!

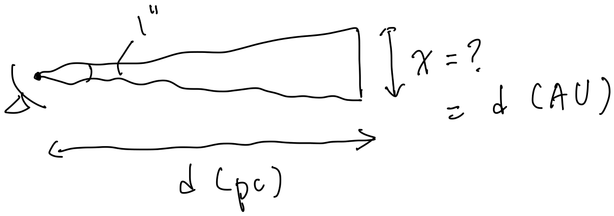


$1\,AU = 1\,\text{pc} \cdot 1''$

$= 1\,\text{pc} \cdot \frac{1}{206265}$ rad

$$\Rightarrow \boxed{1\,\text{pc} = 206265\ AU}$$

useful relation to memorize:



object at d pc $\Rightarrow$ $\underline{1''} \longleftrightarrow \underline{d \text{ AU}}$

3) Virial eq: $2T = -V$

Pleiades cluster: $\begin{cases} \sigma_r \simeq 0.5 \text{ km/s} \\ r_c \simeq 3 \text{ pc} \end{cases}$

$$2 \cdot \frac{3}{2} M \sigma_r^2 = \frac{GM^2}{2\eta r_c} \Rightarrow \boxed{M = \frac{6\eta\sigma_r^2 r_c}{G}}$$

$\underline{\sigma_{radial}}$ $\underset{(3d)}{V_r^2} = 3 \cdot \sigma_r^2$

(i) $\rho(r) = $ const (homogeneous sphere):

last week discussion $\rightarrow$ $PE = -\dfrac{3GM^2}{5r_c}$

$$\updownarrow$$

$$\frac{GM^2}{2\eta r_c}$$

$\Rightarrow \eta = \frac{5}{6} \sim \underline{0.83}$

(ii) Plummer sphere:

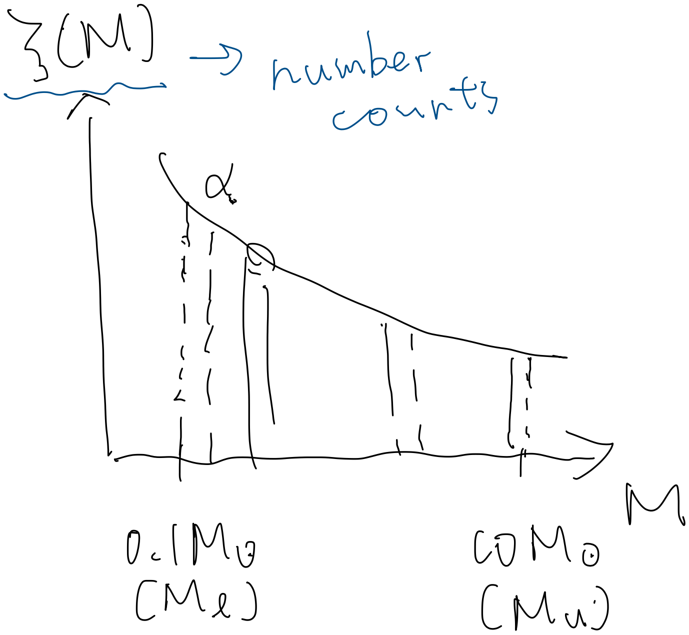
HW 3 & 4 $\rightarrow$ PE, $\eta = \underline{2.64}$

$$\frac{M_{plummer}}{M_{homogeneous}} = \frac{2.64}{0.83} = 3.2 \rightarrow \text{huge difference!}$$

4) Mass functions

$M^{-2.35}\,(dM)$

Salpeter IMF : $\xi(M)\,\Delta M = \xi_0 \left(\frac{\ }{M_\odot}\right) \left(\frac{\ }{M_\odot}\right)$



$$\int \xi(M)\, dM = \xi_0 M^{-2.35}\, dM$$

(M is in $M_\odot$)

M as $\left(\frac{M}{M_\odot}\right)$

$dM \rightarrow d\left(\frac{M}{M_\odot}\right)$

$$N = \int_{M_l}^{M_u} \xi(M)\, dM$$

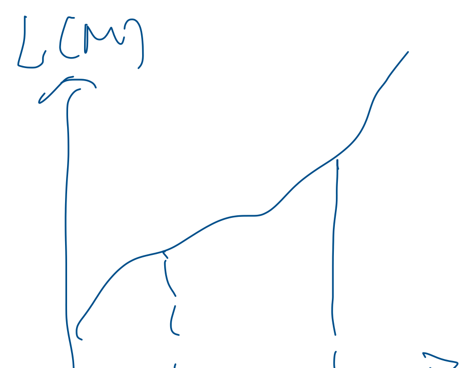
$$\underline{M} = \int_{M_l}^{M_u} \xi(M) \cdot \underline{M}\, dM$$

$$L = \int_{M_l}^{M_u} \xi(M) \cdot \underline{L(M)}\, dM$$

$L(M) \sim M^{-\beta}$

luminosity function

$$= \int_{M_l}^{M_u} \xi(M) \cdot M^{-\beta}\, dM$$




M_ℓ
M_u
M