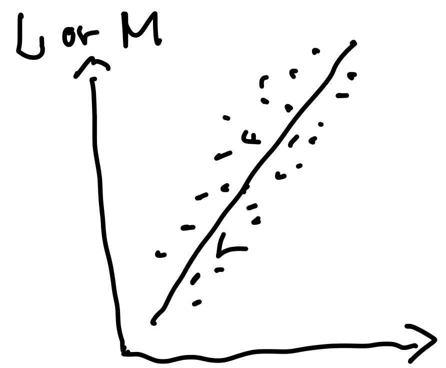
Week 7 Discussion

Wednesday, May 13, 2020 1:59 PM

Tully-Fisher Relation

Strong empirical correlation

$L \propto V_{max}^4$



$\log(W[HI])$

$\sim \log(2 V_{max} \cdot \sin i)$

rotation velocity

① $M(<r) = \dfrac{r V^2(r)}{G}$

$\boxed{\frac{M}{L} \text{ and } I(0) \sim \text{const}}$

$\rightarrow M \propto V_{max}^2 \cdot h_R \longrightarrow L \propto V_{max}^4$

② Lower $I(0)$ in low-surface brightness galaxies

$\rightarrow$ if TF still remains true

$\rightarrow$ higher $\frac{M}{L} \propto \dfrac{1}{\sqrt{I(0)}}$

① $M(<r) = \dfrac{r V^2(r)}{G}$

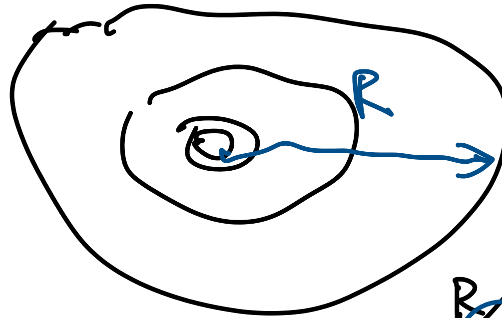
$\Rightarrow$ At $r = h_R$:

$$M(<h_R) = \frac{h_R \cdot V^2(h_R)}{G}$$

$$\Rightarrow h_R \underbrace{V^2(h_R)}_{V_{max}^2} = G \underbrace{M(<h_R)}_{\approx M}$$

$$\Rightarrow \boxed{M \propto V_{max}^2 \cdot h_R}$$

$$\Rightarrow h_R \propto \frac{M}{V_{max}^2} \Rightarrow h_R^2 \propto \frac{M^2}{V_{max}^4} \checkmark$$



$$I(R) = I(0)\, e^{-R/h_R}$$

outside h_R

$\Rightarrow \Sigma(R)$ drop for factor $> e$

Recall HW4 : $L_D = 2\pi I(0) \cdot h_R^2$ ✓

$$\Rightarrow L_D \propto h_R^2 \propto \frac{M^2}{V_{max}^4}$$

If $\frac{M}{L}$ and $I(0) \sim$ const

M

$\Rightarrow L \propto M$

M^2

$$\frac{\ }{L} \sim \text{const} \Rightarrow L \propto M$$

$$M \propto \frac{\mu}{V_{max}^4}$$

$$\Rightarrow \frac{M}{V_{max}^4} \sim \text{const}$$

$$\Rightarrow \boxed{L \propto V_{max}^4}$$

② $I(0) \neq \text{const}$

$$\left\{\begin{array}{l} h_R^2 \propto \dfrac{L}{I(0)} \\[2em] h_R^2 \propto \dfrac{M^2}{V_{max}^4} \end{array}\right. \Rightarrow \frac{M^2}{V_{max}^4} \propto \frac{L}{I(0)}$$

$$\Rightarrow \frac{M}{L} \cdot \frac{M}{\underbrace{V_{max}^4}_{\propto L}} \propto \frac{1}{I(0)}$$

$$\Rightarrow \left(\frac{M}{L}\right)^2 \propto \frac{1}{I(0)}$$

$$\Rightarrow \boxed{\frac{M}{L} \propto \frac{1}{\sqrt{I(0)}}}$$