Week 8 Discussion

Wednesday, May 20, 2020   1:58 PM

① Hot gas & virial temperature

gas atoms σ_r, $\bar{E} = \frac{3}{2}kT$ per particle

Consider ionized H $\rightarrow p^+ + e^-$

atom's K.E. $= \frac{3}{2} m_p \sigma_r^2$ $\xrightarrow{"\!"} p^+$, $\xrightarrow{"\!"} e^-$

E_k for $p^+ = \frac{3}{4} m_p \sigma_r^2$

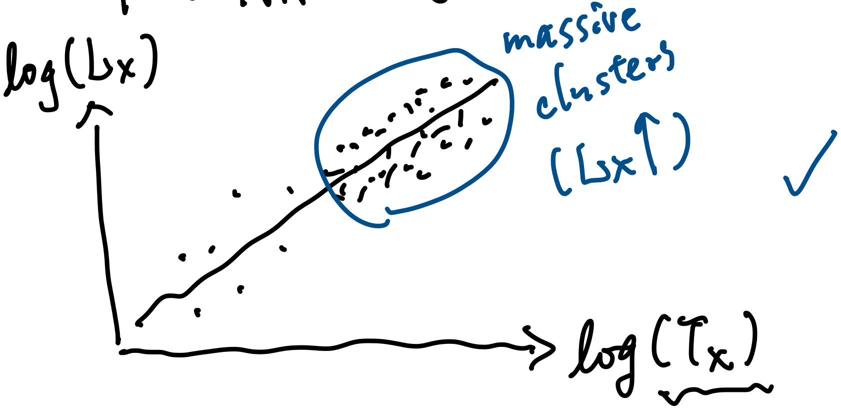
$$\bar{E} = \frac{3}{2}kT = \frac{3}{4} m_p \sigma_r^2 \Rightarrow \boxed{T = \frac{m_p \sigma_r^2}{2k}}$$

Let $\sigma_r = 300\ \mathrm{km/s} = 3\times10^5\ \mathrm{m/s}$

$$\Rightarrow T = \frac{1.67\times10^{-27}(3\times10^5)^2}{2\times1.38\times10^{-23}} = \underbrace{5.44\times10^6}_{T_{vir}}(K)$$

✱ Hot gas in a group or cluster

$T \sim T_{vir} \propto \sigma^2$



$\underbrace{T_x}_{\sim T_{vir}}$

2. $\frac{3}{2}\cancel{m}\sigma^2 = \frac{Gm^{\cancel{2}}}{2\,7r} \Rightarrow m \propto \sigma^2 \cdot r$ $\Big\} \ \underbrace{\sigma^2 \propto r^2}$

$\left\{ m \propto \cancel{\bar{\rho}} r^3 \right.$ $\quad$ const $\quad$ $r \propto \sigma$

$\Rightarrow \boxed{r \propto \sqrt{T}}$

$\therefore m \propto \sigma^2 r \propto \underbrace{T_x \cdot T_x^{\frac{1}{2}}}$

$\propto L^\beta \qquad = T_x^{3/2}$

② Black holes

$\rightarrow$ Schwarzschild radius R_s $\Rightarrow$ $v_{esc} \rightarrow c$

$$\frac{1}{2} m v^2 = \frac{GMm}{R^2} \Rightarrow v = \sqrt{\frac{2GM}{R_s^2}} = c$$

$$\Rightarrow \boxed{R_s = \frac{2GM}{c^2}} \checkmark$$

Earth's mass : 6×10^{24} kg

compress to BH $\Rightarrow R_s = \dfrac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3 \times 10^8)^2}$

$= 9 \times 10^{-3}$ (m)

$\sim$ 1 cm

white dwarf : $R \sim 100$ km

neutron star : $R \sim 1$ km

SMBH in MW $\Rightarrow R_s$?

$M = 4 \times 10^6 M_\odot$

$$\Rightarrow R_s = \frac{2 \times 6.67 \times 10^{-11} \times 4 \times 10^6 \times 2 \times 10^{30}}{(3 \times 10^8)^2}$$

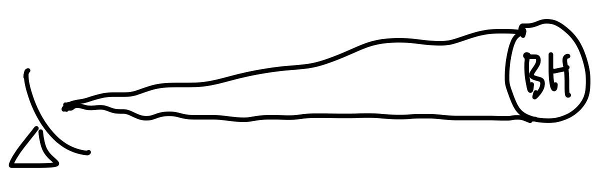
$$\sim 10^{10} \text{ (m)} = 10^7 \text{ (km)}$$

$$\simeq \frac{10^7}{\underbrace{1.5 \times 10^8}_{\text{sun-earth (km)}}} = 0.067 \text{ (AU)}$$

observe from Earth $\Rightarrow$ $d = 8$ kpc

$\Rightarrow 1'' \longleftrightarrow 8000$ AU

resolution required:



$l = 2 \cdot R_s = 0.067 \times 2$

~ 0.1 AU

$$1'' \times \frac{0.1}{8000} = 1.25 \times 10^{-5} \text{ as} = \boxed{12.5 \text{ mas}}$$

Event Horizon Telescope (EHT)

mm/sub-mm $\Rightarrow$ $\sim 1\,\text{mm} \simeq 10^{-3}$ (m)

max baseline ~ earth's diameter

$\simeq 6400\,\text{km} \times 2 \sim 10^{4}\,\text{km}$

$= 10^{7}$ (m)

angular resolution

observed wavelength

$$\theta \sim \frac{\lambda}{B_{max}} \sim \frac{10^{-3}}{10^{7}} \simeq 10^{-10}\ (\text{rad})$$

$$\simeq 10^{-10} \times 206265 \simeq 2.06 \times 10^{-5}$$

$$\sim \boxed{20\ \mu as}$$