Week 2 Discussion

Monday, April 5, 2021 11:48 PM

Cosmological Redshift

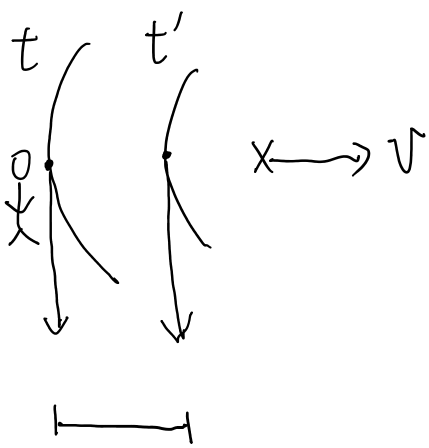
$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{\lambda_{obs}}{\lambda_{em}} - 1$$

$$\Rightarrow 1 + z = \frac{\lambda_{obs}}{\lambda_{em}} \quad \text{or} \quad \boxed{\lambda_{obs} = (1+z)\lambda_{em}} \quad \text{— ①}$$

relation with receding velocity?

→ doppler shift

(i) non-relativistic ($v \ll c$, $z \ll 1$)



$$\lambda_{obs} = cT_{obs} = cT_{em} + vT_{em} \quad \text{— ②} \quad ✓$$

$$= \lambda_{em} + \frac{v}{f_{em}}$$

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$$/\lambda_{em}$$

$$\Rightarrow \frac{\lambda_{obs}}{\lambda_{em}} = 1 + \frac{v}{f_{em} \cdot \lambda_{em}} = 1 + \frac{v}{c}$$

Compare with ①: $\lambda_{obs} = (1+z)\lambda_{em} = (1+\frac{v}{c})\lambda_{em}$

$$\Rightarrow z \approx \frac{v}{c} \Rightarrow \boxed{v \approx cz}$$

higher redshift (e.g. $z = 1.5$)? ✗

(ii) relativistic Doppler shift

(special relativity). $z > 2$

HW1: $z = \sqrt{\frac{1+v/c}{1-v/c}} - 1$

$$\Rightarrow 1 + z = \sqrt{\frac{1+\beta}{1-\beta}} = \frac{\lambda_{obs}}{\lambda_{em}} \quad (\beta \equiv \frac{v}{c})$$

Consider time dilation: $\underbrace{dt}_{\text{rest}} = \gamma \cdot \underbrace{d\tau}_{\text{t' frame}} \quad (\gamma = \frac{1}{\sqrt{1-\beta^2}})$

$\sqrt{1-\beta}$

rest $\qquad$ t' frame

② $\Rightarrow \lambda_{obs} = c\,T_{obs} = c\,T_{em} + v\,T_{em}$

$$= c\gamma\tau + v\gamma\tau$$

$$= \gamma \underbrace{c\tau}_{\lambda_{em}} \left(1 + \underbrace{\frac{v}{c}}_{\beta}\right)$$

$$\Rightarrow \lambda_{obs} = \lambda_{em} \cdot \gamma(1+\beta)$$

$$\frac{1+\beta}{\sqrt{1-\beta^2}} = \frac{\cancel{1+\beta}}{\sqrt{\cancel{(1+\beta)}(1-\beta)}}$$

$$= \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\Rightarrow \lambda_{obs} = \lambda_{em} \cdot \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\Rightarrow \frac{\lambda_{obs}}{\lambda_{em}} = \sqrt{\frac{1+\beta}{1-\beta}} = 1+z$$

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