Week 4 Discussion

Friday, April 23, 2021 3:53 PM

Time / Age

- lookback time

* cosmic expansion $\Rightarrow H(t),\ a(t),\ z(t)$

$\rightarrow H(a),\ H(z)$

$$H(t) = \frac{\dot{a}}{a} = \frac{da}{a\,dt} \Rightarrow dt = \frac{da}{Ha}$$

$$\int_{t_e}^{t_0} dt = \int_{a_e}^{a_0} \frac{da}{Ha} = \int_z^0 \frac{1+z}{H}\, d\left(\frac{1}{1+z}\right)$$

$$= \int_z^0 \frac{-(1+z)(1+z)^{-2}}{H} = \int_0^z \frac{dz}{H(1+z)}$$

$$\Rightarrow t_0 - t_e = \int_0^z \frac{dz}{H(z)(1+z)} \rightarrow \text{lookback time}$$

$$H^2(t) = \frac{8\pi G}{3c^2} \rho(t)$$

For a matter-dominated universe ($\Omega_m = 1$)

$$\rho \ldots \quad \rho_0 (1+z)^3$$

$$\Rightarrow \rho(t) = \rho_0 / a^3 = \rho_0 (1+z)^3$$

$$H^2(t) = H_0^2 / a^3 = H_0^2 (1+z)^3 \Rightarrow \underline{H(z) = H_0 (1+z)^{3/2}}$$

$$t_0 - t_e = \int_0^z \frac{dz}{H_0 (1+z)^{5/2}} = \frac{2}{3H_0} \left[1 - (1+z)^{-\frac{3}{2}} \right]$$

Distances

(1) physical/proper distance

$$ds = c\,dt = a \underbrace{dr}_{\text{co-moving distance}}$$

(2) luminosity distance

observed flux $l = \dfrac{L \rightarrow E/t}{4\pi d_L^2}$

$E/t \cdot A$

$$= \frac{L}{4\pi (a_0 r)^2 (1+z)^2}$$

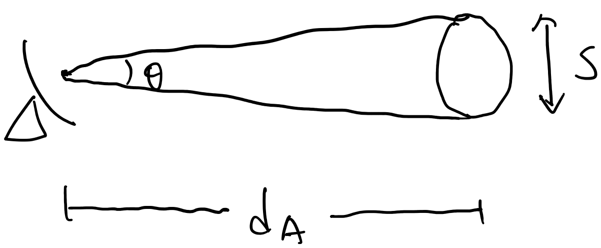
$E = \dfrac{hc}{\lambda}$

① 1

$$\begin{cases} ① \ \dfrac{1}{1+z} : \text{redshift of photon's } \lambda \\ ② \ \dfrac{1}{1+z} : \text{rate of photon emission} \end{cases}$$

$$\Rightarrow \boxed{d_L = \underbrace{a_0}_{=1} r(1+z)}$$

(3) angular diameter distance



$$s = d_A \cdot \theta = a(t) \cdot r \cdot \theta$$

$$\Rightarrow \boxed{d_A = \frac{a_0 r}{1+z} = \frac{d_L}{(1+z)^2}}$$

$$\theta = \frac{\overset{s}{\ell}(1+z)}{r} = \frac{s}{ar}$$