Week 6 Problem Session

Thursday, February 11, 2021 10:56 AM

Midterm Review

1) Luminosity, Flux, Magnitude

2) Angular size $\leftrightarrow$ linear size

3) Virial Theorem

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1) Given Sun: $T_{eff} \approx 5780\,K$

$R_\odot \approx 69500$ km

(i) solar luminosity?

(ii) flux observed from Earth

(iii) If Vega has $\left[\begin{array}{l} L_{vega} = 40\,L_\odot \\ d_{vega} = 7.68\text{ pc} \end{array} \right]$

$\rightarrow$ apparent magnitude of Sun ($m_\odot$)

(iv) absolute magnitude? ($M_\odot$)

(i) $L_\odot$

$$I = \frac{L}{\ldots} = \sigma_{SB} \cdot T_{eff}^4$$

$$I = \frac{L}{4\pi R^2} = \sigma_{SB} \cdot T_{eff}^4$$

energy flux $(E/t \cdot A)$

$R_\odot$: radius

$$\Rightarrow L_\odot = 4\pi R_\odot^2 \cdot \sigma T^4 \simeq 3.85 \times 10^{33}\ erg/s$$

(ii) flux observed from Earth.

$$F_\odot = \frac{L_\odot}{4\pi d^2} = \frac{4\pi R_\odot^2 \cdot \sigma T^4}{4\pi d^2} \simeq 1.37 \times 10^6\ erg/s \cdot cm^2$$

d: 1AU

(iii) $\begin{cases} L_{vega} = 40 L_\odot \\ d_{vega} = 7.68\ pc \end{cases} \Rightarrow m_\odot = ?$

① $m_1 - m_2 = -2.5 \log\left(\frac{F_1}{F_2}\right)$ → compare two objects with different flux

② $m - M = 5 \log(d) - 5$

m, M: same object; d: (pc)

$$① \Rightarrow m_\odot - m_{vega} = -2.5 \log\left(\frac{L_\odot}{4\pi d_\odot^2} \cdot \frac{4\pi d_{vega}^2}{L_{vega}}\right)$$

d_{vega}: 7.68 pc

m_{vega}: "

$4\pi d_\odot^2$ L_{vega}

= 0 $\quad$ 1AU $\quad$ $40 L_\odot$

→ pc?

$\Rightarrow m_\odot \approx -27.75$ mag

(−26.8)

→ actual mag. varies by band.

(iv) $M_\odot$

$m_\odot - M_\odot = 5\log(d_\odot) - 5$

$m_\odot$: (iii) $\quad$ $d_\odot$: 1AU → pc

$= \frac{1}{206265}\,pc \simeq 4.85\times10^{-6}\,pc$

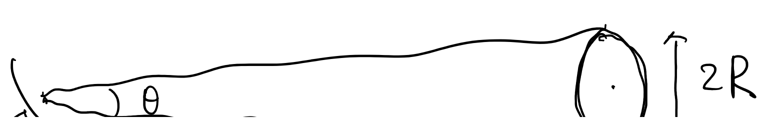
$\Rightarrow M_\odot \simeq 6$ mag

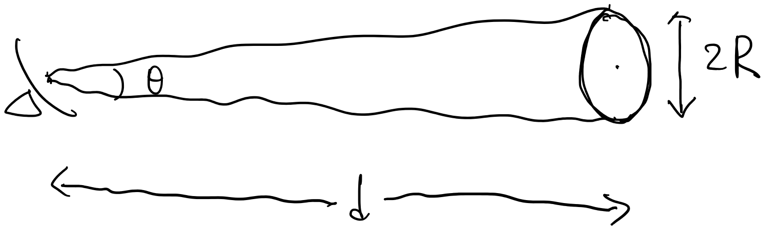
(4.86)

2) Angular sizes of [Sun / Moon] as viewed from Earth.

* objects far way

→ small angle approximation.

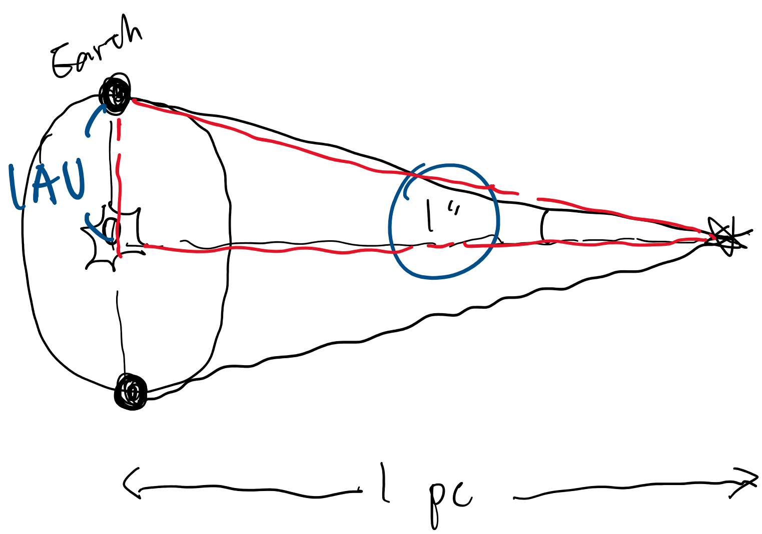




$$2R = d \cdot \theta \Rightarrow \boxed{\theta = \frac{2R}{d}}$$

$$\begin{cases} R_\odot = 695700 \text{ km} \\ d_\odot = 1.5 \times 10^8 \text{ km} \end{cases} \qquad \begin{cases} R_{moon} = 1737 \text{ km} \\ d_{moon} = 384{,}399 \text{ km} \end{cases}$$

$$\begin{cases} \theta_\odot = 0.01 \text{ rad} \approx 35' \\ \theta_{moon} = 0.01 \text{ rad} \approx 35' \end{cases}$$ total eclipses are possible!



$$1 \text{ AU} = 1 \text{ pc} \cdot 1''$$

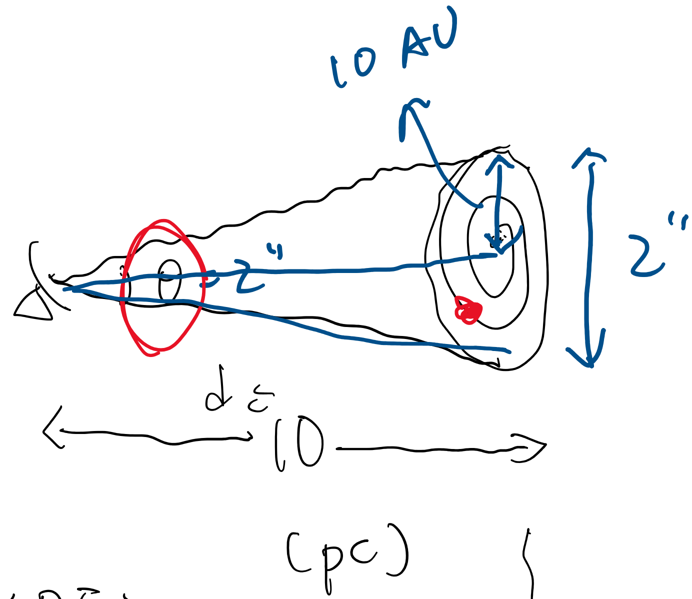
$$= 1 \text{ pc} \cdot \frac{1}{206265} \text{ rad}$$

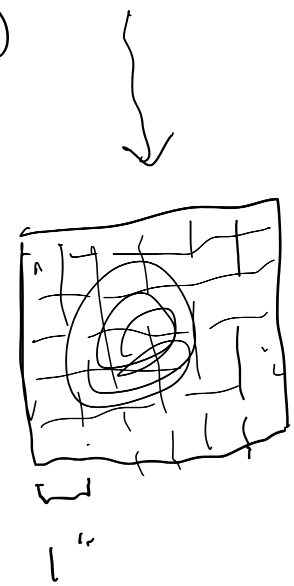
$$\Rightarrow \boxed{1\,pc = 206265\ AU}$$

useful relation :

object at d pc

$\Rightarrow$ 1'' $\longleftrightarrow$ d AU





3) Virial eq : $2\langle KE\rangle = -\langle PE\rangle$

σ_r : radial velocity dispersion

r_c : core radius.

isotropic motion

$$2 \cdot \frac{3}{2} M \sigma_r^2 = \frac{GM^2}{2\eta r_c}$$

$$\Rightarrow \boxed{M_{vir} = \frac{6\eta\sigma_r^2 r_c}{G}}$$

η → depends on density profile.

(i) $\rho(r) = \text{const}$ (homogeneous sphere)

last week : $PE = -\frac{3GM^2}{5 r_c}$

$5 r_c$

$\updownarrow$

$$-\frac{GM^2}{2\eta r_c} \Rightarrow \eta = \frac{5}{6} \sim 0.83$$

(ii) Plummer sphere :

HW 3 & 4 : PE $\Rightarrow \eta = 2.64$

$$\frac{M_{plummer}}{M_{homogeneous}} = \frac{2.64}{0.83} = 3.2$$

huge difference!