Week 7 Problem Session

Thursday, February 18, 2021 10:50 AM

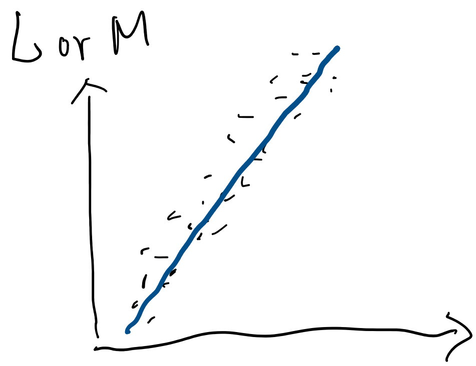
Tully-Fisher Relation

Strong, empirical correlation

$L \propto V_{max}^4$

$\downarrow$

distance ladder



$\log(W[HI])$

$\sim \log(2\underbrace{V_{max}}_{\text{rotational velocity}} \sin i)$

① $M(<r) = \dfrac{rV^2(r)}{G}$

$$\boxed{\frac{M}{L} \text{ and } I(0) \sim \text{const}}$$

$\rightarrow M \propto V_{max}^2 \cdot h_R \longrightarrow L \propto V_{max}^4$

Scale length

② Lower $I(0)$ in low-surface brightness galaxies

→ if TF relation still remains true

→ higher $\frac{M}{L} \propto \frac{1}{\sqrt{I(0)}}$

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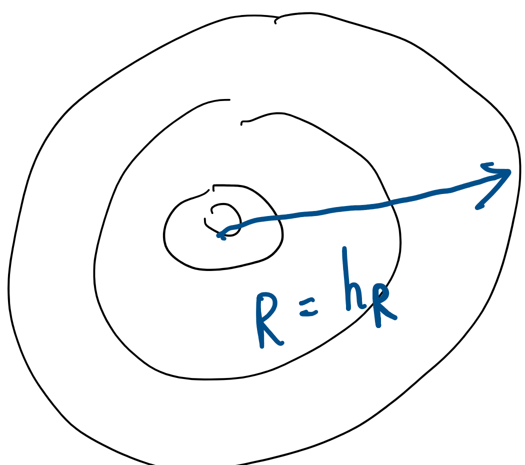
① $M(<r) = \frac{r V^2(r)}{G}$

⇒ At $r = h_R$

$$M(<h_R) = \frac{h_R \cdot V^2(h_R)}{G}$$

$$\Rightarrow h_R \underbrace{V^2(h_R)}_{V_{max}^2} = G \underbrace{M(<h_R)}_{\simeq M}$$



$$I(R) = I(0) e^{-R/h_R}$$

outside h_R

⇒ $\Sigma(R)$ drop for

$\Rightarrow \boxed{M \propto V_{max}^2 \cdot h_R}$

$\Rightarrow \Sigma(R)$ drop for factor $> e$

$$\Rightarrow h_R \propto \frac{M}{V_{max}^2} \Rightarrow h_R^2 \propto \frac{M^2}{V_{max}^4}$$

Recall HW4: $L_D = 2\pi I(0) \cdot h_R^2$

$$\Rightarrow L_D \propto h_R^2 \propto \frac{M^2}{V_{max}^4}$$

If ① $I(0) \sim$ const

② $\frac{M}{L} \sim$ const $\Rightarrow L \propto M$

$$M \propto \frac{M^2}{V_{max}^4}$$

$$\Rightarrow \frac{M}{V_{max}^4} \sim \text{const}$$

$$\Rightarrow \boxed{L \propto V_{max}^4}$$

② $I(0) \neq \text{const}$

low surface-brightness $\rightarrow I(0) \downarrow$

$$\begin{cases} h_R^2 \propto \dfrac{L}{I(0)} \\ h_R^2 \propto \dfrac{M^2}{V_{max}^4} \end{cases} \Rightarrow \frac{M^2}{V_{max}^4} \propto \frac{L}{I(0)}$$

$$\Rightarrow \frac{M}{L} \cdot \frac{M}{V_{max}^4} \propto \frac{1}{I(0)}$$

$\propto L$

$$\Rightarrow \left(\frac{M}{L}\right)^2 \propto \frac{1}{I(0)} \Rightarrow \frac{M}{L} \uparrow \propto \frac{1}{\sqrt{I(0)}} \downarrow$$