Week 8 Problem Session

Thursday, February 25, 2021 10:49 AM

Two topics:

① Hot gas (X-ray)

② Black Holes

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① Hot gas & virial temperature

gas atoms σ_r, $\bar{E} = \frac{3}{2} kT$ per particle.

Consider ionized $H \rightarrow p^+ + e^-$

atom's $E_k = \frac{3}{2} m_p \sigma_r^2$ $\rightarrow p^+$, $\rightarrow e^-$

E_k for $p^+ = \frac{3}{4} m_p \sigma_r^2$

$$\bar{E} = \frac{\cancel{3}}{\cancel{2}} kT = \frac{\cancel{3}}{\cancel{4}_{\,2}} m_p \sigma_r^2 \Rightarrow \boxed{T_{vir} = \frac{m_p \sigma_r^2}{2k}}$$

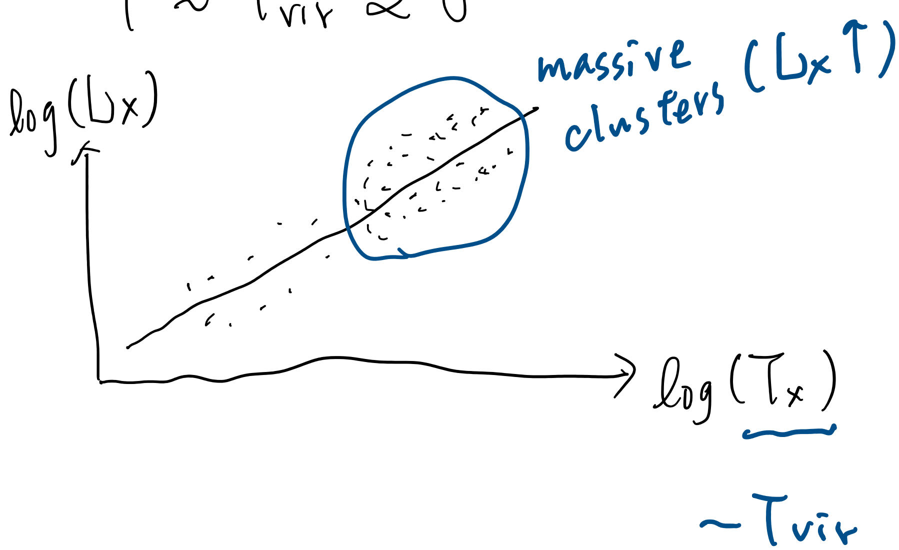
Let $\sigma_r = 300 \, km/s = 3 \times 10^5 \, m/s$

$$= \frac{1.67 \times 10^{-27} (3 \times 10^5)^2}{\ldots} \quad 6$$

$$\Rightarrow \underline{\underline{T_{vir}}} \simeq \frac{1.67\times10^{-27}(3\times10^{5})^{2}}{2\times1.38\times10^{-23}} \simeq \underline{5.44\times10^{6}\ (K)}$$

☆ Hot gas in galaxy clusters

$$T \sim T_{vir} \propto \sigma^{2}$$



$$2\cdot\frac{3}{2}\cancel{m}\sigma^{2} = \frac{Gm^{\cancel{2}}}{2\eta r} \Rightarrow \begin{cases} m \propto \sigma^{2}\cdot r \\ m \propto \underbrace{\rho}_{\text{const}}\cdot r^{3} \end{cases}$$

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$$\Rightarrow \sigma^{2} \propto r^{2} \Rightarrow r \propto \sigma \propto \sqrt{T}$$

$$\therefore m \propto \sigma^{2}\cdot r \propto T_x \cdot T_x^{\frac{1}{2}} = T_x^{3/2}$$

$$\therefore \underline{\underline{m}} \propto \sigma^2 \cdot r \propto T_x \cdot T_x^2 = \underline{\underline{T_x^{\,\prime 2}}}$$

$$\propto L^{\beta}$$

$$\Rightarrow L = T_x^{\underline{\gamma}}$$

$$\Rightarrow \log L \sim \underline{\underline{\gamma}} \cdot \log T_x$$

② Black Holes

→ Schwarzschid radius $R_s \Rightarrow V_{esc} \to c$

$$\frac{1}{2} m v^2 = \frac{GMm}{R} \Rightarrow V_{esc} = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow c = \sqrt{\frac{2GM}{R_s}} \Rightarrow \boxed{R_s = \frac{2GM}{c^2}}$$

<u>Earth's</u> mass: 6×10^{24} kg

Compress to BH $\Rightarrow R_s = \dfrac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3 \times 10^8)^2}$

$$= 9 \times 10^{-3} \text{ (m)}$$

$\sim 1\ \mathrm{cm}$

neutron star : $R \sim 1\ \mathrm{km}$

white dwarf : $R \sim 100\ \mathrm{km}$

SMBH in MW center $\Rightarrow R_s$?

$$M = 4 \times 10^6 M_\odot$$

$$\Rightarrow R_s = \frac{2 \times 6.67 \times 10^{-11} \times 4 \times 10^6 \times 2 \times 10^{30}}{(3 \times 10^8)^2}$$

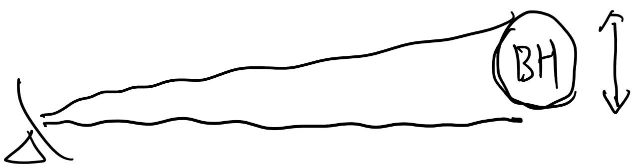
$$\sim 10^{10}\ (\mathrm{m}) = 10^7\ (\mathrm{km})$$

$$= \frac{10^7}{\underbrace{1.5 \times 10^8}_{\text{sun-earth (km)}}} = 0.067\ (\mathrm{AU})$$

observe from Earth $\Rightarrow d = 8\ \mathrm{kpc}$

$\Rightarrow 1'' \Leftrightarrow 8000\ \mathrm{AU}$

resolution required:



$$l = 2 \cdot R_s = 0.067 \times 2$$

$$\sim 0.1\ \mathrm{AU}$$

$$1'' \times \frac{0.1}{8000} = 1.25 \times 10^{-5}\ (\mathrm{as}) = \boxed{12.5\ (\mathrm{mas})}$$

Event Horizon Telescope (EHT)

mm / sub-mm $\Rightarrow \sim 1\ \mathrm{mm} = 10^{-3}\ (\mathrm{m})$

max baseline $\sim$ earth's diameter

$$= 6400\ \mathrm{km} \times 2 \sim 10^4\ \mathrm{km}$$

$$= 10^7\ (\mathrm{m})$$

$$\theta \sim \frac{\lambda}{B_{max}} \sim \frac{10^{-3}}{10^7} = 10^{-10}\ (\mathrm{rad})$$

$$= 10^{-10} \times 206265 = 2.06 \times 10^{-5}$$

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$$\sim \boxed{20 \ \mu as}$$